The Secretary Problem with Independent Sampling

José Correa[1], Andrés Cristi[1], Laurent Feuilloley[2], Tim Oosterwijk[3], Alexandros Tsigonias-Dimitriadis[4]

[1]Universidad de Chile, [2]Université Lyon 1, [3]Vrije Universiteit Amsterdam, [4]Technical University of Munich

Background

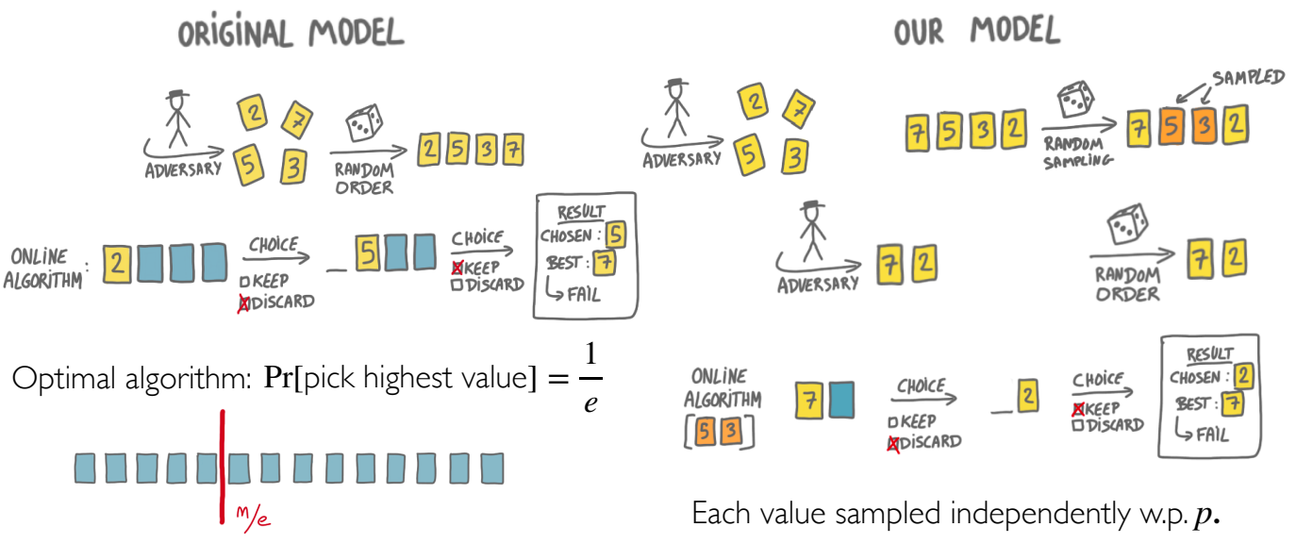
→ The secretary problem is probably the most well-studied optimal stopping problem with many applications.

→ One of its limitations for modeling real-world situations is the assumption that the online values are completely unknown. On the other hand, assuming that a distribution from where the values are drawn is fully known might be too optimistic.

→ Particularly when unexpected events might happen, even assuming a distribution from which we can sample might be strong. We want to combine the idea of having samples representing past data with having arbitrary values chosen adversarially.

→ We propose a robust framework that
1) incorporates past experience
2) is more general than explicitly assuming a distribution
3) does not take as input the exact number of elements arriving.

A new model with sampling



Optimal algorithm: $\Pr[\text{pick highest value}] = \frac{1}{e}$

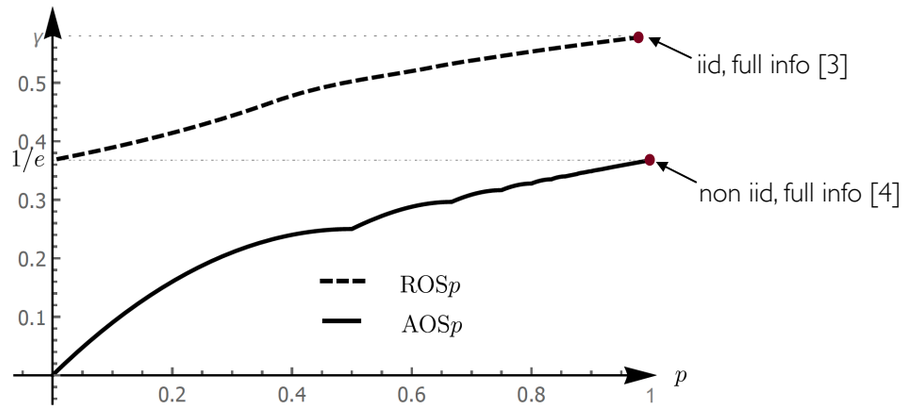
For *adversarial* order, not much we can do!

Each value sampled independently w.p. p.

Objective: $\max \Pr[\text{pick highest online value}]$.

Results

We obtain best possible algorithms for both orders and for ***any*** value of p.



Adversarial order

Single-threshold algorithm depending only on p.
Prove optimality: Transform problem & combinatorial arguments on a conflict graph.

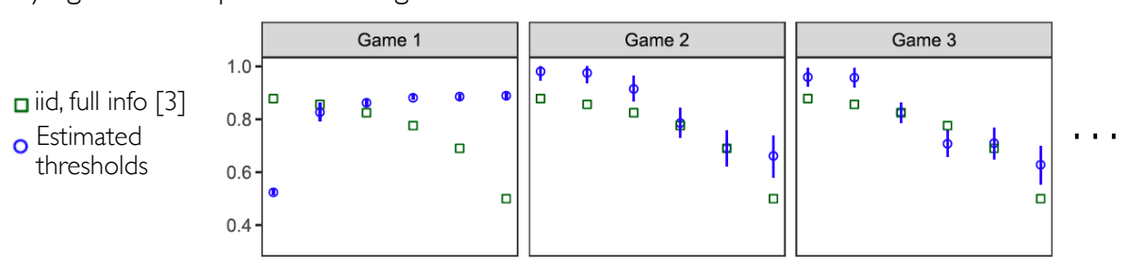
Random order

Fixed sequence of decreasing thresholds in time.

Extensions and applications

→ A convenient framework to study combinatorial problems with additional information (e.g., online matchings [1], matroid secretary problems).

→ The thresholds for the random order seem to nicely capture the strategies that agents develop when playing repeatedly the secretary problem with an unknown distribution [2]. Ongoing work: Can we provide a different explanation than [2], namely that players are playing close to optimal all along ?



Data from seven-box games in [2]

[1] Kaplan, H., Naori, D., Raz, D, Online Weighted Matching with a Sample, SODA 2022.
[2] Goldstein G.D., McAfee P.R., Suri S., Wright J.R., Learning when to stop searching, Management Science, 2020.
[3] Gilbert J., Mosteller F., Recognizing the maximum of a sequence, Journal of the American Statistical Association, 1966.
[4] Esfandiari, H., HajiAghayi, M., Lucier, B., Mitzenmacher, M. Prophets, secretaries, and maximizing theprobability of choosing the best, AISTATS 2020.